Introduction to Rank-width

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Third Workshop on Graph Classes, Optimization, and Width Parameters

Outline

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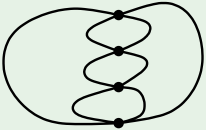
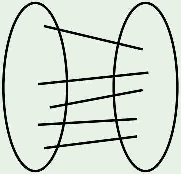
Connectivity

Partition (E, F) of $E(G)$:



$v(X)$ =#vertices meeting both X and $E \setminus X$.

Partition (E, F) of $V(G)$:



$e(X)$ =#edges meeting both X and $V \setminus X$.

M: matroid, $\lambda(X) = r(X) + r(E(M) - X) - r(E(M))$.

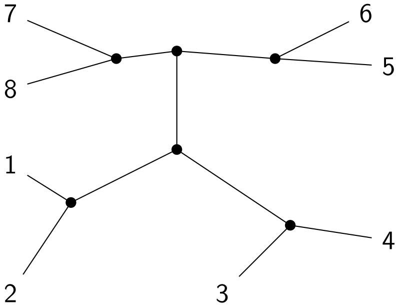
A function $f : 2^V \to \mathbb{Z}$ is a connectivity function if

(i) $f(X) + f(Y) \geq f(X \cap Y) + f(X \cup Y)$, (submodular)

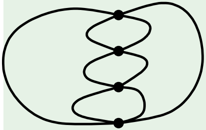
(ii) $f(X) = f(V \setminus X)$, (symmetric)

(iii) $f(\emptyset) = 0$.

Branch-decomposition of a connectivity function f: a pair (T, L) of a *subcubic tree* T and a *bijection* $L : V \longrightarrow \{\text{leaves of T}\}$.

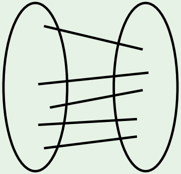


Branch-width



$V = E(G)$

Carving-width



$V = V(G)$

Branch-width of matroids
(Branch-width of λ) $+ 1$.
$\lambda(X) =$
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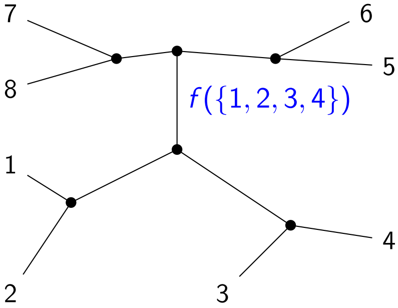
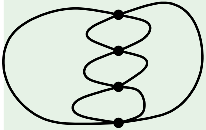


Width of an edge e of T: $f(A_e)$
(A_e, B_e) is a partition of V given by deleting e.

Branch-width



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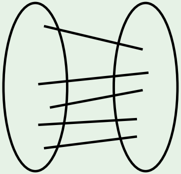

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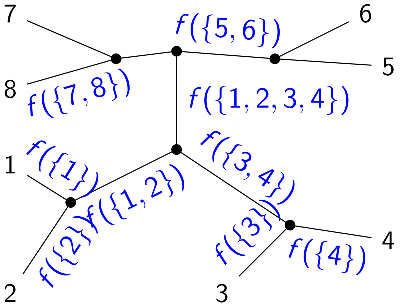

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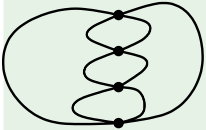
Width of (T, L): $\max_e \text{width}(e)$

Branch-width



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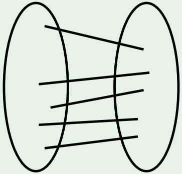

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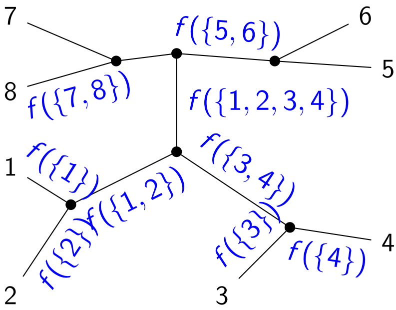
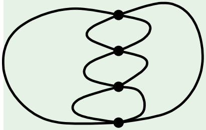


Width of an edge e of T: $f(A_e)$
(A_e, B_e) is a partition of V given by deleting e.

Width of (T, L): $\max_e \text{width}(e)$

Branch-width: $min_{(T,L)}$ width(T, L).
(If $|V| \leq 1$, then branch-width$=0$)

Branch-width



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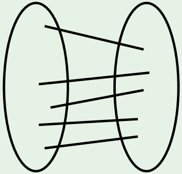

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Branch-width is "good"

Testing Branch-width $\leq k$ for fixed k

- Branch-width of graphs: Linear (Bodlaender, Thilikos '97)
- Carving-width of graphs: Linear (Thilikos, Serna, Bodlaendar '00)
- Branch-width of matroids represented over a fixed finite field: $O(|E(M)|^3)$ (Hliněný '05)
- Any connectivity function: $O(\gamma n^{8k+6} \log n)$ (Oum and Seymour '07)

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Finding Branch-decomposition of width $\leq k$ for fixed k

- Branch-width of graphs: Linear (Bodlaender, Thilikos '97)
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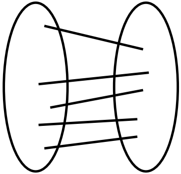
- Branch-width of matroids represented over a fixed finite field: $O(|E(M)|^3)$ (Hliněný and Oum '07)
- Any connectivity function: $O(\gamma n^{8k+9} \log n)$ (Oum and Seymour '07)

Cut-rank function: another connectivity function

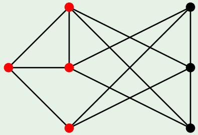
$(X\quad,\quad Y)$: partition of $V(G)$



$$\rho_G(X) = \text{rank} \begin{array}{cc} & Y \\ X & \left(\boxed{\text{0-1 matrix}}\right) \end{array}$$

(The matrix is over the binary field $\mathrm{GF}(2)$.)

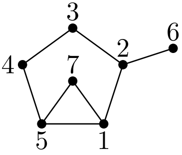
$$\rho(\text{red vertices}) = \text{rank} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} = 2.$$



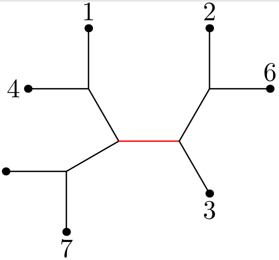
Rank-width

Definition of Rank-width

Rank-width of a graph G = Branch-width of the cut-rank function ρ_G



Graph



Rank-decomposition
Width= 2

Rank-width: min width(rank-decomposition).

Find a rank-decomposition of width $\leq k$ for fixed k

$O(n^3)$ (Hliněný and Oum '07)

Clique-width
Courcelle, Engelfriet, and Rozenberg '93 / Courcelle, Olariu '00

- *k*-expression: algebraic expression on vertex-labelled graphs with k labels $1, 2, \ldots, k$.
  - $\cdot_i$ a single vertex with label i
  - $G_1 \oplus G_2$ disjoint union
  - $\rho_{i \to j}(G)$ relabel vertices of label i into j
  - $\eta_{i,j}(G)$ $(i \neq j)$ add edges vertices of label i and j
- Clique-width of a graph G:
  min k such that G has a k-expression.

$$G_1 = \eta_{1,2}(\cdot_1 \oplus \cdot_2)$$

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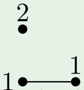

$G_2 = \rho_{2\to 1}(G_1) \oplus \cdot_2$

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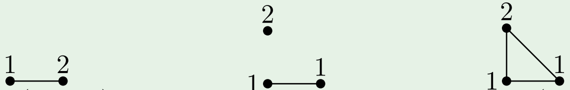


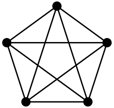
$G_1 = \eta_{1,2}(\cdot_1 \oplus \cdot_2)$ $G_2 = \rho_{2 \to 1}(G_1) \oplus \cdot_2$ $G_3 = \eta_{1,2}(G_2)$

Rank-width and clique-width are 'equivalent' (OS. '06)

$$\mathrm{rwd}(G) \leq \mathrm{cwd}(G) \leq 2^{\mathrm{rwd}(G)+1} - 1.$$

More Examples

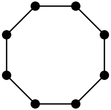
- For $n \geq 2$,
  $\mathrm{cwd}(K_n) = 2$,
  $\mathrm{rwd}(K_n) = 1$.



K_5

- $\mathrm{cwd}(C_n) = \begin{cases} 2 & \text{if } n = 3, 4 \\ 3 & \text{if } n = 5, 6 \\ 4 & \text{if } n \geq 7. \end{cases}$

  $\mathrm{rwd}(C_n) = \begin{cases} 1 & \text{if } n = 3, 4, \\ 2 & \text{if } n \geq 5. \end{cases}$



C_8

- Trivial Upper Bound ($n =$ number of vertices)

$$\mathrm{rwd}(G) \leq \lceil n/3 \rceil$$

Outline

Solvable problems when rank-width is bounded (I)

Courcelle, Makowsky, and Rotics '00

Every graph problem
expressible in *monadic second-order logic formula*
is solvable in time $O(n^3)$
for graphs having rank-width at most k for fixed k.

CMR'01: Counting the number of true assignments in polynomial time.

Can I find a partition of vertices into three subsets such that each set has no edges inside? (graph 3-coloring problem)

$$\begin{aligned}\exists X_1 \exists X_2 \exists X_3 \forall v \forall w (v, w \in X_1 \Rightarrow \neg \operatorname{adj}(v, w))\\ \wedge \forall v \forall w (v, w \in X_2 \Rightarrow \neg \operatorname{adj}(v, w))\\ \wedge \forall v \forall w (v, w \in X_3 \Rightarrow \neg \operatorname{adj}(v, w)) \cdots\end{aligned}$$

Solvable problems when rank-width is bounded (II)

Many other problems (that are not MS_1 expressible) can be also solved in polynomial time for graphs of bounded rank-width.

- Finding a chromatic number. (Kobler and Rotics '03)
- Deciding whether a graph has a Hamiltonian cycle. (Wanke '94)
- Given a monadic second-order logic formula φ, list all m such that there is a partition $(X_1, \ldots, X_m)$ of $V(G)$ such that $\varphi(X_i)$ is satisfied for all i. (Rao '07)

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Not all problems are easy on graphs of bounded clique-width:
Gurski and Wanke '06: found a problem that is

- solvable in linear time on clique-width ≤ 2
- but NP-complete on clique-width ≤ 6.

Outline

Rank-width-preserving operations

Or, Cut-rank preserving operations

- Local complementation at v:
  Replace the subgraph induced by neighbors of v with its complement graph.

$$\text{rwd}(G * v) = \text{rwd}(G).$$

- Pivoting on an edge $e = uv$:
  Let X: common neighbors of u and v, Y: neighbors of u but non-neighbors of v, Z: non-neighbors of u but neighbors of v.
  1. "Toggle" adjacency between X and Y, between Y and Z, and between Z and X.
  2. Swap the labels of u and v.

  In fact, $G \wedge uv = G * u * v * u$.

$$\text{rwd}(G \wedge xy) = \text{rwd}(G).$$

Graph relations

Induced Subgraphs

- (Deletion)*
- $\mathrm{cwd}(H) \le \mathrm{cwd}(G)$ (CO '00) $\mathrm{rwd}(H) \le \mathrm{rwd}(G)$
  if H is an induced subgraph of G.

Vertex-minors

- (Deletion + Local Complementation)*
- $\mathrm{rwd}(H) \le \mathrm{rwd}(G)$ if H is a vertex-minor of G.

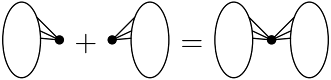
Pivot-minors

- (Deletion + Pivoting)*
- $\mathrm{rwd}(H) \le \mathrm{rwd}(G)$ if H is a pivot-minor of G.

Graph compositions for rank-width

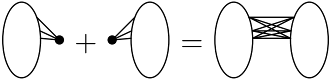
Can we glue graphs maintaining small rank-width?

- Disjoint union
- 1-sum



Rank-width=Max Rank-width of Blocks

- 1-join



Rank-width=Max rank-width of prime induced subgraphs (wrt split decomposition)

('1-sum' is a special case of a '1-join'.)

Basic graph classes

- Clique-width≤ 2 : cographs (P_4-free graphs)
- Rank-width ≤ 1: distance-hereditary graphs (O. '05)
  (every connected induced subgraph has the same distance function.)
  Golumbic and Rotics '00: clique-width of distance-hereditary graphs ≤ 3

Forbidden vertex-minors

- Bouchet'87,'88: distance-hereditary if and only if no C_5 vertex-minor.

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- O. '05: Graphs of rank-width $\leq k$ = graphs with forbidden pivot-minors having at most $(6^{k+1} - 1)/5$ vertices.
- O. '05: More generally, well-quasi-ordering theorem:
  Graphs of bounded rank-width are well-quasi-ordered by pivot-minors.

In every infinite sequence of graphs $G_1, G_2, \ldots$ of bounded rank-width, there exist $i < j$ such that G_i is isomorphic to a pivot-minor of G_j.

Equivalently, if X is a set of graphs of bounded rank-width closed under taking pivot-minors, there is a finite list of graphs $G_1, \ldots, G_m$ such that X contains a graph isomorphic to G if and only if none of $G_1, \ldots, G_m$ is isomorphic to a pivot-minor of G.

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This well-quasi-ordering theorem implies...

- Graphs of bounded tree-width are well-quasi-ordered by minors. (Robertson and Seymour '90)

Outline

Relation to Tree-width / Branch-width of graphs

- Corneil and Rotics '05: $\text{cwd}(G) \le 3(2^{\text{twd}(G)-1})$.
- Kanté '07: $\text{rwd}(G) \le 4\,\text{twd}(G) + 2$.

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Let $I(G)$ be the graph obtained by subdividing each edge.
We prove the following:

Theorem

- $\mathrm{rwd}(I(G)) = \mathrm{bwd}(G)$ or $\mathrm{rwd}(I(G)) = \mathrm{bwd}(G) - 1$, unless $\mathrm{bwd}(G) = 0$.
- G is a vertex-minor of $I(G)$.
- The line graph $L(G)$ is a vertex-minor of $I(G)$.

- $\mathrm{rwd}(G) \le \mathrm{bwd}(G) \le \mathrm{twd}(G) + 1$.
- $\mathrm{rwd}(L(G)) \le \mathrm{bwd}(G) \le \mathrm{twd}(G) + 1$.

Binary Matroids

$M = (E, I)$ is a matroid if

- E is a finite set,
- I is a collection of independent subsets of E, satisfying
  - (I1) $\emptyset \in I$
  - (I2) If $A \in I$ and $B \subseteq A$, then $B \in I$.
  - (I3) For every $Z \subseteq E$, the maximal subsets of Z in I have the same size $= r(Z)$, the rank of Z.

We call M a binary matroid
if $\exists$ matrix N over $\mathrm{GF}(2)$ such that

- E: set of column vectors of N,
- $I = \{X \subseteq E : X \text{ is independent as vectors}\}$.

Matroid connectivity: $\lambda(X) = r(X) + r(E \setminus X) - r(E) + 1$.

Standard Representation of Binary Matroids

- Elementary row operations don't change the binary matroid.
- We may delete dependent rows.

Therefore, we may assume that binary matroids are represented by matrices of the following form $\left(\begin{array}{c|c} \text{identity} & B \\ \text{matrix} & \end{array} \right)$.

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Fundamental graph

A bipartite graph whose adjacency matrix is $\begin{pmatrix} 0 & B \\ B^t & 0 \end{pmatrix}$.

Pivoting: By elementary row operations, we may swap one column in the identity matrix with another column in B.

Matroid minor operations on standard representation

- Pivoting.
- Delete a column not in the identity matrix.
- Delete a row.

Bipartite Graphs and Binary Matroids

- Connectivity of Binary Matroids = Cut-rank of its Fundamental (Bipartite) Graph $+1$
- $\mathrm{bwd}(M) = \mathrm{rwd}(G) + 1$
  if M is a binary matroid and G is a fundamental graph of M.

Cycle Matroids $M(G)$

The cycle matroid $M(G)$ of a graph G is a binary matroid on $E(G)$ represented by $\left(\begin{array}{c|c} \text{identity matrix} & B \end{array} \right)$ where B is defined as follows: (Assume G is connected.)

- Let $T =$ edge-set of a spanning tree.
- $B_{ef} = 1 \Leftrightarrow e \in T, f \notin T$, and $T - e + f$ is a spanning tree.

Fundamental Graph of a graph G

The fundamental graph $F(G, T)$ of G with respect to T is a bipartite graph on $E(G)$ such that

$$e, f \text{ are adjacent} \Leftrightarrow T - e + f \text{ is a spanning tree.}$$

Hicks, McMurray Jr '07. Indepdently Mazoit, Thomassé (submitted)

If G has a cycle of length ≥ 2, then $\text{bwd}(G) = \text{bwd}(M(G))$.

$\mathrm{rwd}(I(G)) = \mathrm{bwd}(G)$ or $\mathrm{bwd}(G) - 1$

- Let $G' = G +$ (apex vertex v) (v is adj to all vertices of G.)
- Let T be the edge set of G' consisting of all edges incident with v.

Observation

$I(G)$ is equal to the fundamental graph of G' with respect to T.

Remember?:

Fundamental Graph of a graph G

The fundamental graph $F(G, T)$ of G with respect to T is a bipartite graph on $E(G)$ such that

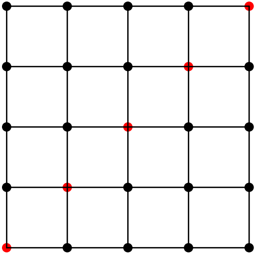
$$e, f \text{ are adjacent} \Leftrightarrow T - e + f \text{ is a spanning tree.}$$

Consequently: $\mathrm{rwd}(I(G)) = \mathrm{rwd}(F(G, T)) = \mathrm{bwd}(M(G')) - 1 = \mathrm{bwd}(G') - 1 = \mathrm{bwd}(G)$ or $\mathrm{bwd}(G) - 1$.

Outline

What is the Rank-width of the $n \times n$ grid?

- Clique-width of the $n \times n$ grid $= n + 1$ if $n \geq 3$. (Golumbic, Rotics '00)
- Rank-width $\leq n - 1$.



How to show the lower bound for rank-width?

Tangle

Tangle of a connectivity function f

An f-tangle $\mathcal{T}$ of order k is a set of subsets of V satisfying the following three axioms:

(T1) $f(X) < k \Rightarrow X \in \mathcal{T}$ or $V \setminus X \in \mathcal{T}$.

(T2) If $X, Y, Z \in \mathcal{T}$, then $X \cup Y \cup Z \neq V$.

(T3) $V \setminus \{v\} \notin \mathcal{T}$ for all $v \in V$.

Robertson and Seymour. GM X

$bwd(f) \geq k$ if and only if there exists an f-tangle of order k.

(T2) can be replaced with the following two axioms:

(T2a) If $B \subseteq A$, $A \in \mathcal{T}$, and $\rho_G(B) < k$, then $B \in \mathcal{T}$.

(T2b) If $A, B, C \in \mathcal{T}$ are disjoint sets, then $A \cup B \cup C \neq V$.

$\mathrm{rwd}(n \times n \text{ grid}) \geq \lceil 2n/3 \rceil$

- P_i: vertices in the i-th row ($i = 1, 2, \ldots, n$)
- Q_i: vertices in the i-th column ($i = 1, 2, \ldots, n$)

Our approach is similar to Kleitman and Saks (in GM X).

Two Lemmas

Let X be a subset of V such that $\rho(X) < n - 1$.

- $P_i \subseteq X$ for some i if and only if $Q_j \subseteq X$ for some j.
- $P_i \not\subseteq X$ for all i if and only if $P_r \subseteq X^c$ for some r.

Say X is small if $\rho(X) < \lceil 2n/3 \rceil$ and $P_i \not\subseteq X$ for all i.
Let $\mathcal{T}$ be a set of small sets.
Then (T1), (T2a), (T3) are easy.

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Open problems

- Let f be a connectivity function. Prove one of the following.
  1. There exists a FPT algorithm to test branch-width$\leq k$.
  2. To test branch-width $\leq k$, in the worst case, at least $n^{O(\log k)}$ queries are necessary.
- Prove that the rank-width of the $n \times n$ grid is $n-1$.
  Say X is small if $\rho(X) < \lceil 2n/3 \rceil$ and $P_i \not\subseteq X$ for all i.
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Thanks for the attention!